Optical Guiding in a Free-Electron Laser

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By use of two-dimensional approximations for the equations that describe a high-gain free-electron laser (FEL) amplifier, and the properties of optical fibers, it is shown that the coherent interaction between the light and the electron beam in a FEL can optically guide the light. In the exponential-gain regime, the FEL performance in the presence of strong diffraction can be simply described by a cubic equation for the complex gain and the dispersion relation for an optical fiber. The phenomenon of optical guiding is illustrated with two-dimensional numerical simulations. The phenomenon has applications to short-wavelength FEL’s, to directing of intense light, and to bending of x rays.

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It has long been known that the coherent interaction between the light and the electron beam in a free-electron laser (FEL) produces a phase shift of the light, and that the sign of the effect can be such that the light is refracted toward the electron beam. The electron beam in a high-gain FEL bunches on an optical wavelength; because of the bunching, the beam has an effective index of refraction greater than unity. This is in sharp contrast to the behavior of an unmagnetized (and unbunched) plasma, and is the basis for the optical guiding effects described here. The effect occurs even if there is negligible growth in light intensity and can be important in many situations.

The usual analysis of step-profile optical fibers assumes that the fiber consists of a central core of radius a and index of refraction n, and a cladding of index n_{cl}. In our treatment, the core is the electron beam and the cladding is free space, and so n_{cl} = 1. We can make the assumption that the fiber is weakly guiding:

\[ |n - 1| \ll 1. \]  

This inequality is quite good for all cases of interest, and is consistent with the assumption of a slowly varying phase \( \phi \) of the optical field,

\[ \frac{d\phi}{dz} \ll k = \omega/c, \]  

familiar from FEL theory.

Following Marcus,\(^3\) we consider guided modes with only one transverse electric field component \( E_x \) (but both magnetic and electric longitudinal components), so that \( E_x \) varies as \( J_n(\kappa r) \) inside the fiber and as \( H_n^1(\gamma r) \) outside, where \( J_n \) and \( H_n^1 \) are Bessel functions and Hankel functions of the first kind, respectively. The arguments of the functions are \( \kappa = (n^2k^2 - \beta^2)^{1/2} \), \( \gamma = (\beta^2 - k^2)^{1/2} \), and the field is assumed to vary as \( \exp[i(\beta z - \omega t)] \). Continuity of \( B_z \) and \( E_z \) at the fiber edge yields the dispersion relation:

\[ \kappa J_{n+1}(\kappa a) = \frac{\gamma K_{n+1}(\gamma a)}{K_n(\gamma a)}, \]  

with

\[ (\kappa^2 + \gamma^2)a^2 = V^2(n^2 - 1)k^2a^2. \]  

The quantity \( V \) is called the "fiber parameter."

The condition for mode cutoff in a fiber is \( \gamma \rightarrow 0 \). While formally there is no cutoff for the mode \( LP_{01} \) (the first index labels the Bessel function, the second labels the zeros), which is the dominant mode for FEL amplification, it is incorrect to think of the mode as bound by the fiber for all \( V > 0 \) since it extends far outside the beam for \( V << 1 \). For the mode \( LP_{01} \) to be considered guided, we somewhat arbitrarily require that the 1/e point of \( E_x \) be within 5 times the fiber radius. This condition corresponds to demanding that \( V^2 > 1 \).

The analysis can be extended to a fiber with gain (or loss) by permitting \( n \) to be complex. The dispersion relation, Eq. (3), is unchanged, but \( \kappa \) and \( \gamma \) can now also be complex. From numerical solution of the complex dispersion relation, we find that the above criterion generalizes to

\[ \text{Re}(V^2) + |\text{Im}(V^2)|/2.4 > 1. \]  

The index of refraction of an optically bunched beam comes from the FEL equations as formulated by Prosnitz, Szoek, and Neil:

\[ \text{Re}(n) - 1 = \frac{1}{k} \frac{d\phi}{dz} = \frac{2\pi \varepsilon_0 \omega}{mc^3k} \left( \frac{\cos \psi_i}{\gamma_i} \right), \]  

\[ \text{Im}(n) = \frac{1}{k} \frac{d\psi}{dz} = \frac{2\pi \varepsilon_0 \omega}{mc^3k} \left( \frac{\sin \psi_i}{\gamma_i} \right). \]  

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TABLE I. Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current ($I$)</td>
<td>270 A</td>
</tr>
<tr>
<td>Electron beam radius in the wiggler ($a$)</td>
<td>0.01 cm</td>
</tr>
<tr>
<td>Electron Lorentz factor ($\gamma_0$)</td>
<td>2000</td>
</tr>
<tr>
<td>Fractional electron energy spread ($\Delta \gamma/\gamma$)</td>
<td>$1.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>Laser wavelength ($2\pi/k$)</td>
<td>2500 Å</td>
</tr>
<tr>
<td>Rayleigh length ($z_r$)</td>
<td>50 cm</td>
</tr>
<tr>
<td>Dimensionless rms wiggler vector potential ($a_w$)</td>
<td>4.35</td>
</tr>
<tr>
<td>Wiggler length ($L$)</td>
<td>30 m</td>
</tr>
<tr>
<td>Wiggler period ($2\pi/k_w$)</td>
<td>10 cm</td>
</tr>
<tr>
<td>Input laser power ($P_i$)</td>
<td>30 Mw</td>
</tr>
<tr>
<td>Output laser power ($P_o$)</td>
<td>585 MW</td>
</tr>
</tbody>
</table>

In Eqs. (6) and (7), $e_z$ and $a_w$ are the normalized rms amplitudes, $e_z = e |E_z|/\sqrt{2mc^2}$ (for a linear wiggler), and $a_w = e |B_w|/\sqrt{2k_w mc^2}$, where $k_w$ is the wiggler wave number. The wave number of the optical field is $k$, the current density is $j$, $\psi_j$ is the phase of an electron in the ponderomotive potential well, and $\gamma_j$ is its Lorentz factor. The angular brackets denote an average over the electron distribution. We use Gaussian cgs units.

Numerical simulations were performed with the two-dimensional (2D) FEL code FRED. The code follows an axisymmetric laser beam around an electron beam that bunches longitudinally (in $\psi$). Axisymmetric diffraction effects are fully included, via the paraxial wave approximation; refractive and gain effects are included through the local source terms provided by the electron beam. The parameters of one simulation are listed in Table I. Figure 1 is a three-dimensional contour plot of laser intensity, while Fig. 2 presents some details of the simulation.

In the exponential-gain regime we can go further in the analysis by extending the linear analysis of Bonifacio, Pellegrini, and Narducci to include the effects of diffraction. To do so, we write the usual longitudinal electron equations derived by Kroll, Morton, and Rosenbluth in complex form. The particle motion equations are unchanged, but the complex field equation has a transverse gradient term added:

$$\frac{\partial e_z}{\partial z} = \frac{2\pi ie_z a_w}{mc^3} f_B \sum_j \frac{e^{-i\psi_j}}{\gamma_j} + \frac{i}{2k} \nabla^2 e_z,$$

where $e_z$ is now a complex field amplitude. The total number of electrons is $N$, and $f_B$ is the difference of

FIG. 1. A three-dimensional plot of laser intensity vs $r$ and $z$ inside the wiggler for case I. Note that the laser profile is nearly constant for 60 Rayleigh lengths.
central energy of the electron beam, and

\[ A = \frac{2\pi e I a_0^2 f_B}{mc^3} z_r^2 \]

(11)

is the dimensionless parameter that measures coupling between the electron beam and the light.

The expression for the fiber parameter \( V \) of the electron beam in terms of \( \gamma \) is

\[ V^2 = \left(2ka^2/z_r\right)(1 + \gamma). \]

(12)

For the parameters of the simulation, the cubic yields \( V_2 = 1.01 - 0.12i, \lambda = 0.01 - 0.12i \). Our criterion for guiding Eq. (5) is satisfied, although the laser beam is somewhat more tightly confined to the electron beam than \( |V| = 1 \) would predict. In terms of \( V \), the discrepancy is only about 20%.

The general procedure for evaluating the importance of the guiding of laser light by an electron beam is iterative. The cubic, Eq. (10), is solved with an assumed value for \( \kappa \). From the solution for \( \lambda \), Eq. (12) gives \( V \). The value of \( V \) determines, through Eqs. (3) and (4), new values for \( \gamma \) and \( \kappa \). Iteration produces a consistent solution for the laser beam size and the growth rate, if a guided solution exists. Thus, in the exponential growth regime we have a complete analytic solution of the problem and there is no need to employ the 2D numerical code.

Because of optical guiding one can contemplate very long FEL’s. In this way, it appears possible to have a small electron-beam radius and a very long wiggler (hence a very-high-gain FEL) even in the vacuum ultraviolet (vuv) range.

Because of the effect of optical guiding it is possible to direct and focus the FEL-generated optical beam. This is of interest for very intense beams, such as are contemplated for laser inertial fusion, where lenses and mirrors of conventional materials would be destroyed by the light. Use of optical guiding appears to be relatively straightforward since a simple magnetic deflection of the electron beam will result in a deflection of the light.

It should be noted that optical guiding applies also, to very-short-wavelength light. Application of this to the vuv and to soft x rays, which do not interact coherently with normal material, would appear to make possible some interesting devices.

Optical guiding will be effective in an inverse free-electron laser (IFEL) as well as in an FEL,\(^8\) and hence can be important in the operation of an IFEL. Finally, we note that optical guiding may make possible resonant ring FEL’s.\(^8\)

A more extensive treatment of this topic is given by Scharlemann, Wurtele, and Sessler.\(^9\) After the completion of this work our attention was drawn to work by Moore which nicely complements that presented here.\(^10\)
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7A. Gaupp, private communication.
8J. D. Dawson, private communication.
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