Abstract

General considerations of the requirements for a high energy linear collider (here, to be specific, taken to be 1 TeV on 1 TeV) are applied to free-space laser and laser/plasma accelerators. It is shown that the requirements impose very severe constraints upon the new accelerators—so severe, that it seems unlikely that these necessary criteria can be met in the foreseeable future.

1 INTRODUCTION

Over the last two decades much work has been done -- both theoretically and experimentally -- on the acceleration of particles by means of lasers. Considerable progress has been made in the theoretical understanding of the complicated phenomena associated with the interaction of lasers and plasmas, in numerical modeling of such phenomena, in experimental studies, and in the development of lasers. It is correct to characterize the field as having advanced very significantly during the last decades.

Part of the motivation for the study of laser acceleration has been the application of this method to high energy linear colliders. There are, of course, other applications, but the possible use of these new techniques in high energy physics remains a major interest, and motivating force, for the effort. It is this application that we want to examine in the present report.

We consider, to be specific, a 1 TeV x 1 TeV linear collider. Actually, such a device can, most likely, be achieved with "conventional technology", or with only slightly non-conventional technology such as that in the two-beam accelerator, but higher energy is even more daunting.

In this Report we delineate, in Section II, general considerations having to do with luminosity, beamstrahlung, and wall-plug power. This allows us to derive diverse parameters of the accelerated beam. In Section III we consider acceleration in a vacuum. In Section IV we consider plasma accelerators which consist either of acceleration in a uniform plasma or acceleration in a plasma channel. Finally, in Section V we present a discussion.

2 COLLIDER REQUIREMENTS

The requirements of high energy physics are, for our purposes, simply summarized by the luminosity and the beamstrahlung energy spread of the device. Expressions for these quantities are well-known [1] and are:

\[ L = \frac{\frac{1}{2} N^2}{4 \pi \sigma_y^* R} \]  
\[ \delta = \frac{0.88 r_e N^2}{R^2 \sigma_y^* \sigma_z} \]  

where \( f \) is the effective frequency of the collider (rep. rate x bunch number), \( N \) is the number of particles in a bunch, \( \sigma_y^* \) is the rms transverse dimension at the interaction point, \( R \) is the ratio of horizontal to vertical size of the beam, \( r_e \) is the classical particle radius, \( \sigma_z \) is the longitudinal rms dimension of the bunch, and the beam energy is characterized by \( \gamma \). The collider requires an operating power which is simply:

\[ P_b = 2 \gamma mc^2 N f = \eta P_w \]  

where the efficiency \( \eta \) is defined in terms of the wall-plug power, \( P_w \).

Since \( \gamma, L, \) and \( \delta \) are given by the high energy physics requirements and \( P_w \) is given by economic considerations, it is convenient to express the unknown quantities, \( N, \sigma_y^*, \sigma_z, R, \) and \( f \), in terms of the independent quantities. We find, having put in convenient units:

\[ N \sigma_z \left[ \text{cm}^{-1} \right] = 6.4 \times 10^9 \left( \frac{\delta \eta P_w (\text{GW}) R}{L 10^{35} E (\text{TeV})} \right) \]  
\[ f [\text{MHz}] = 0.05 \left( \frac{L 10^{35} E (\text{TeV})}{\delta \sigma_z \left[ \text{cm} \right] R} \right) \]  
\[ \sigma_y^* \left[ \text{nm} \right] = 126 \sqrt{\delta \sigma_z \left[ \text{cm} \right]} \left( \frac{\eta P_w (\text{GW})}{L 10^{35} E^{3/2} (\text{TeV})} \right) \]
In these expressions $\sigma_z$ must scale roughly with the acceleration wavelength $\lambda_{acc}$ (either the laser wavelength in a vacuum acceleration or the plasma wavelength in a plasma accelerator).

### Table 1

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Laser Plasma</th>
<th>Vacuum Laser</th>
<th>Vacuum Laser</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$ (TeV)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$L_0$ (cm$^2$)</td>
<td>$10^{35}$</td>
<td>$10^{35}$</td>
<td>$10^{35}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$n_{pw}$ (GW)</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_{x}$ (µm)</td>
<td>10</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Then</td>
<td>$R$</td>
<td>$100$</td>
<td>$100$</td>
</tr>
<tr>
<td>$N_{\sigma_x}$ (cm$^{-1}$)</td>
<td>$1.3 \times 10^{10}$</td>
<td>$1.3 \times 10^{10}$</td>
<td>$1.3 \times 10^{12}$</td>
</tr>
<tr>
<td>$N_{\sigma_z}$ (cm$^{-1}$)</td>
<td>$1.3 \times 10^{7}$</td>
<td>$1.3 \times 10^{7}$</td>
<td>$1.3 \times 10^{7}$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>$f$ (MHz)</td>
<td>$4.9$</td>
<td>4.9</td>
</tr>
<tr>
<td>Then</td>
<td>$\sigma_{x}$ (cm$^{-1}$)</td>
<td>$1.1 \times 10^{17}$</td>
<td>---</td>
</tr>
<tr>
<td>$\sigma_{y}$ (cm)</td>
<td>$10^{-4}$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta E_{\text{pl}}$ (m)</td>
<td>$2.3 \times 10^{10}$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\Delta E_{\text{pl}}$ (m)</td>
<td>$1.3 \times 10^{-10}$</td>
<td>$1.3 \times 10^{-10}$</td>
<td>$1.3 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

Thus vacuum acceleration (for highly relativistic particles) inevitably involves structures (diaphragms) to deflect the laser light and these structures are subject to material damage.[4] In addition, at high beam intensities there are significant wake field effects due to these diaphragms. The wakes are both longitudinal (limiting the acceleration gradient) and transverse (limiting the bunch transverse size).

A representation of one section of a conceptual vacuum accelerator is shown in Fig 1. The sections are necessary so as to have continuous acceleration; that is, to deflect the laser beam, or introduce a new beam with proper phase, when the particle and the light are no longer in accelerating phase. The diaphragm produces a longitudinal wake which means:

$$\Delta E = \frac{2N_{\text{re}}}{\sqrt{\frac{\pi \sigma_z}{\sigma_y}}} \left( \frac{a}{mc^2} \right)^2$$

where $\Delta E$ is the energy loss from one diaphragm, $a$ is the radius of the hole in the diaphragm, $l$ is the length of a section, and $\sigma_y$ is the transverse rms of the electron beam at the diaphragm. The quantity $\sigma_y$ scales with the laser wavelength, the quantity $a$ is related to the length of a section of the plasma, $\sigma_z$ scales with the laser wavelength and $N_{\sigma_z}$ is fixed by collider considerations. In addition, the diaphragm produces a transverse wake and material damage of the diaphragm restricts the power of the laser.

![Figure 1. Conceptual picture of one period of a vacuum laser accelerator.](image)

### 3 VACUUM ACCELERATORS

There are two general types of vacuum accelerators. The first uses external magnetic fields, or pulses of laser power, to provide transverse bending of particles and, therefore, provides continuous acceleration by the laser as in an Inverse Free-Electron Laser.[2] Inevitably, because of the particles bending, they will radiate and they radiate more and more as their energy is increased. Hence there will be some upper limit on energy at which the synchrotron radiation just compensates the laser energy. Typically that energy is below 1 TeV and, therefore, this class of accelerators is not of interest for high energy physics.

Acceleration in a vacuum is greatly restricted by the Lawson-Woodward theorem. [3] That theorem states that there is no net energy gain by a particle provided:

1. The region of interaction extends over all space,
2. The laser fields are vacuum fields; i.e., there are no nearby walls,
3. The particle is highly relativistic; i.e., its velocity is essentially unchanged (and $c$),
4. No static fields are present (so motion is in a straight line)
5. Nonlinear effects are negligible.

### 4 PLASMA ACCELERATORS

Plasma accelerators reach high gradients by employing a drive laser pulse or pulses to create a large amplitude plasma wave. Laser-plasma interactions limit the transverse size of the drive laser pulse to be of order the $c/\omega_p$. In this circumstance the accelerating field created in a homogeneous plasma will typically have transverse gradients of order $\omega_p/cE_x$ and a transverse component of order $n_{\omega_p}/E_x$, where $E_x$ is the accelerating field. For high gradients, the plasma focusing will dominate any external focusing. Thus the accelerating structure, i.e., the plasma, controls both transverse and longitudinal motion of the accelerated bunch. Alternatively, plasma waves can be driven by a particle bunch, which approach has problems of its own that are not addressed here.

The transverse gradients of the accelerating field, and the strength of the focusing, can be reduced by accelerating in a plasma channel.[5] The channel also acts as an optical fiber, guiding the laser light and allowing for acceleration lengths much greater than a diffraction length. In the
hollow plasma channel, the transverse focusing force is reduced by a factor of order $\gamma_0^{-2}$, where $\gamma_0$ is the group velocity of the laser in the plasma channel. The beam radius then increases as $\gamma_0^{-1/2}$.

The misalignment of a quadrupole in a conventional linac corresponds to, in a plasma accelerator, a stray electric field in the plasma. The strength of the accelerating field will determine the transverse focusing strength and thereby the level to which stray fields need to be controlled. These undesirable fields can originate from laser-plasma instabilities, density inhomogeneities, etc. An estimate of the focusing strength in the plasma gives an equilibrium radius in the plasma of $r_e=(\varepsilon B_e)^{1/2}$.

The electron beam in the plasma will generate wakes. The higher order mode losses and transverse BBU for the accelerated bunch in the plasma will set constraints on the accelerated current. These effects should be less pronounced in a plasma than in vacuum laser accelerators since the plasma accelerator has a longer wavelength (and hence wider transverse dimensions) and the plasma is, ideally, homogeneous in the longitudinal direction.

5 DISCUSSION

We now examine the results of collider designs based on the above constraints, as seen in Table 1. We have, arbitrarily, taken the beam power to be 20 MW so that the wall plug power, $P_W$, depending on the efficiency of the laser-to-particle system, is in the range of hundreds of megawatts which would seem to be about all one could afford. The interesting range of accelerating wavelength can be covered with only two cases, the first--with $\lambda_{ACC} = 100 \mu$m--corresponds to a plasma accelerator and the second--with $\lambda_{ACC} = 1 \mu$m--corresponds to a vacuum accelerator. There really is very little freedom in these choices; they correspond to reasonable plasma densities and reasonable wavelengths for powerful lasers. The choice of accelerating wavelength sets the scale for $\sigma_z$; we have chosen it to be 1/10 of the wavelength.

Column 1 of the Table gives parameters for a plasma accelerator and column 2 gives parameters for vacuum accelerators. As can be seen from the first two columns of the Table, obtaining the desired high energy physics puts very severe requirements on the laser-driven accelerators. Firstly, the repetition rate must be very high and, secondly, the emittance (and consequently the beam size) must be very small. The emittance is two orders of magnitude smaller and the assumed $\beta$ values are one or three orders of magnitude smaller, than are planned for future linear colliders. The first is a technological problem (but not a trivial problem) and the second puts very tight requirements upon component tolerance and component location. Our calculation has been classical, but at the very small $\sigma_y$ we are considering QED corrections are important even at 1 TeV. Quantum corrections change $\delta$ in column 1 of the Table to .02, and $\delta$ in column 2 from 0.1 to 0.003. Operation with large quantum effects brings in other complications. Changing parameters so that quantum effects are small furthermore forces one into a regime where wake fields become important.

Perhaps one can argue that beamstrahlung is not really a constraint. After all, the concept of plasma compensation [6] has been put forward (but never experimentally established). In order to study that case we present, in column three of Table 1, a situation with vacuum acceleration but beamstrahlung a factor of 100 greater than in the first two columns. The laser accelerator is now less difficult than before (the emittance and associated tolerances are now achievable), but still very difficult to achieve.[7] We are now in a regime where longitudinal wakes are important---with a staging length $\leq 500 \mu$m the decelerating gradient is 1 GV/m.

We have presented an overall review of the requirements that high energy colliders put upon any laser-driven accelerator. We find that the 1 TeV application is daunting enough, and the application to still higher energy is even more difficult.

Perhaps, it can be argued, imposing collider requirements upon laser accelerators, at a time when the accelerators are still in their infancy, is unfair. On the other hand, serious work on laser accelerators has been in progress for 15 years and, furthermore, often claims are made that seem to ignore--rather than consider and address--the elementary constraints discussed in this paper. We believe that it is important that advocates of laser-driven accelerators, for collider applications, face up to the limits discussed in this paper.

Of course there are other applications of laser-driven accelerators--and many of these are "easier" to achieve that are colliders. These, alone, justify, in our opinion, further work on the subject.

We wish to thank Glen Westenskow for his valuable discussions.

6 REFERENCES